shapes which are transformable into perfectly regular boundaries. The object is to transform to boundaries which may be nonregular in the mathematical sense (i.e., no exact closed form solution exists), but which are smooth and slowly varying, so that elementary numerical solutions may be applied to give highly accurate results. Since singularities and sharp corners have been eliminated, good results are obtained from numerical techniques using relatively coarse meshes. The method should be used in conjunction with the network analog theory described in [9], which shows how to avoid slowly convergent and uncertain iterative numerical techniques.

The references cited by Laura appear to be concerned mainly with the solution of more difficult field theory problems, e.g., determination of the cut off frequencies of waveguides of unusual cross section [12]–[16]. As Dr. Laura states, Laplace's equation is invariant under a conformal transformation, but this is not the case for more general wave equations. Here conformal transformations may be applied to transform the boundaries into regular shapes, but the field equations become far more complex. In a sense one is transforming one type of complex problem into another, transferring the difficulty from the boundary to the form of equation. A good example of this is described in [14], where the groove guide is transformed into a parallel-plate guide filled with a nonuniform anistropic medium. In solving these transformed problems, complicated Fourier or integral equation techniques need to be employed. The methods have proven quite successful in many instances, but may be considered to be rather specialized.

I would like to take this opportunity to make a correction to eq. (24) of [1] which should read

$$C_g + 2C_f = C_I. \tag{24}$$

REFERENCES

[1] E. Goursat, *Bull. Soc. Math.* (France), vol. 26, p. 206, 1898.
[2] N. I. Muschelisvili, *Math. Ann.*, vol. 107, pp. 282–312, 1932.
[3] H. B. Wilson, "A method of conformal mapping and the determination of stresses in solid propellant rocket grains," rep S-38, Rohm and Haas Company, Huntsville, AL, 1963
[4] P. A A. Laura, "A survey of modern applications of the method of conformal mapping," Revista de la Unión Matemática Argentina, vol. 27, pp. 167–179, 1975.
[5] ——, "Conformal mapping of a class of doubly connected regions," The Catholic Univ., of America, Washington, DC, Tech. rep. 8 to NASA, Jan. 1965.
[6] M. K. Richardson, "A numerical method for the conformal mapping of finite, doubly connected regions with application to the torsion problem for hollow bars," Clemson Univ., SC, 1965.
[7] D. C. Ives, "A modern look at conformal mapping including multiply connected regions," *American Inst. of Aeronautics and Astronautics J.*, vol. 14, no 8, pp. 1006–1011, Aug. 1976.
[8] N. D. Halsey, "Potential flow analysis of multielement airfoils using conformal mapping," *American Inst. of Aeronautics and Astronautics J.*, vol. 17 (12), pp. 1281–1289, Dec. 1979.
[9] C. Caratheodory, *Conformal Representation*. Cambridge at the University Press, 1932, p. I.
[10] P. A. A. Laura, "A survey of modern applications of the method of conformal mapping," *Revista de la Unión Matemática Argentina*, vol. 27, pp. 167–179, 1975.
[11] H. H. Meinke, K. P Lange, and J. F Ruger, "TE- and TM-waves in waveguides of very general cross section." *Proc. IEEE*, vol. 51, pp. 1436–1443, Nov. 1963.
[12] F. J. Tischer, "Conformal mapping in waveguide considerations," *Proc. IEEE*, vol. 51, pp. 1050–1051, July 1963.
[13] F. J. Tischer, "The Grove Guide, a low-loss waveguide for millimeter waves," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 291–296, Sept. 1963.
[14] M. Chi and P. A A. Laura, "Approximate method of determining the cut-off frequencies of waveguides of arbitrary cross section," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 248–249, Mar. 1964.
[15] P. A. A. Laura, "Conformal mapping and the determination of cut-off frequencies of waveguides with arbitrary cross-section," *Proc. IEEE*, vol. 54, pp. 1078–1080, Aug. 1966.
[16] G. P. Bava and G. Perona, "Conformal mapping analysis of a type of groove guide," *Electron. Lett.*, vol. 2, pp. 13–15, Jan. 1966.
[17] R. Levy, "Direct non-iterative numerical solution of field theory problems having irregular boundaries using network analogs," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 596–604, June 1980.

Comments on "The Matched Feedback Amplifier: Ultrawide-Band Microwave Amplification with GaAs MESFET's"

DOUGLAS J. H. MACLEAN

In the above paper [1], the bandwidth of a single GaAs MESFET was extended by means of series inductances (L_D and L_{FB}) in the feedback path.

The authors used voltage vector diagrams to illustrate the action of this feedback and that of a simpler circuit in which L_D and L_{FB} of Fig. 1(a) were both set to zero, leaving only R_{FB} between drain and gate. A polar plot of the feedback current for $L_D = 0.6$ nH and $L_{FB} = 0.45$ nH [c.f., 1, fig. 2 curve D] was also shown [1, fig. 4].

The behavior of their circuit diagram [1, fig. 1(a)], (reproduced here as Fig. 1), can perhaps be better understood if conventional loop gain plots, rather than their vector diagrams, are used. Loop gain is a well-known concept in feedback amplifiers, and can be expected to be familiar to readers of the paper [1]. The appropriate point at which to break the loop is in the branch which contains R_{FB}. If the loop gain was to be measured by means of a 50-Ω network analyzer, R_{FB} could be replaced by two resistors of some 80-Ω connected to the input and output of the analyzer, to obtain the best accuracy in measuring the quantities S_{11} and S_{22}. This was also done for the present analysis, and the resulting open-loop gains computed, corresponding to the authors' curves A and D [1, fig. 2]. These loop gains are shown in Fig. 2. If we examine Fig. 2 it is clear that the above values of L_D and L_{FB} have kept the magnitude of the loop gain more nearly constant. The phase of this loop gain goes through 0° just above 15 GHz (compared with 13.75 GHz cited), at which point the "feedback" is purely positive, but less than -15 dB. In common parlance, the gain margin is some 15 dB, and the gain margin frequency around 15 GHz. A phase margin cannot be defined since the magnitude of the loop gain never exceeds one in the frequency range shown. Comparing Fig. 2 with [1, fig. 4] it is evident that the phase angles of the loop gain and of the feedback current differ by some 180°. Finally, because of the low values of the loop gain (LG), the quantity $|1 - LG|$ is close to unity, and there can be little gain enhancement due to "positive feedback."

An improved method of assessing feedback, especially in the frequency range covered by this amplifier has recently been described [2]–[5]. It is known as the "embedding network" method, and is mainly intended for multiloop amplifiers characterized from s-parameter measurements, which yield the best, commercially obtainable accuracy. A circuit diagram can, however, be used with some loss of realism arising from the artificiality of most circuit diagrams at such frequencies.

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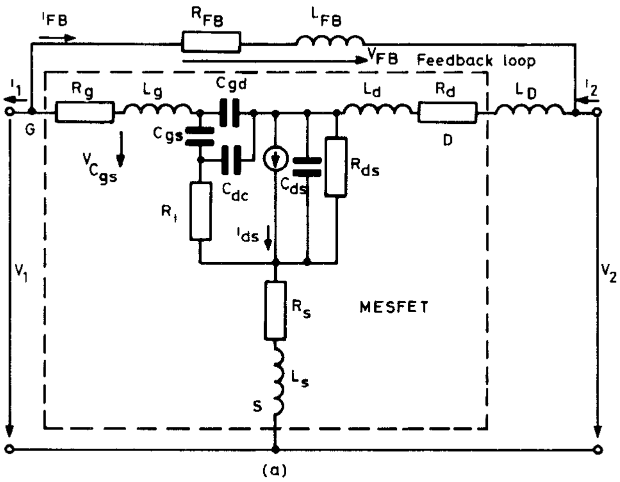

Fig. 1. High-frequency circuit diagram of the basic feedback amplifier (note: $i_{ds}=g_m V(C_{gs})$) [1, fig. 1(a)].

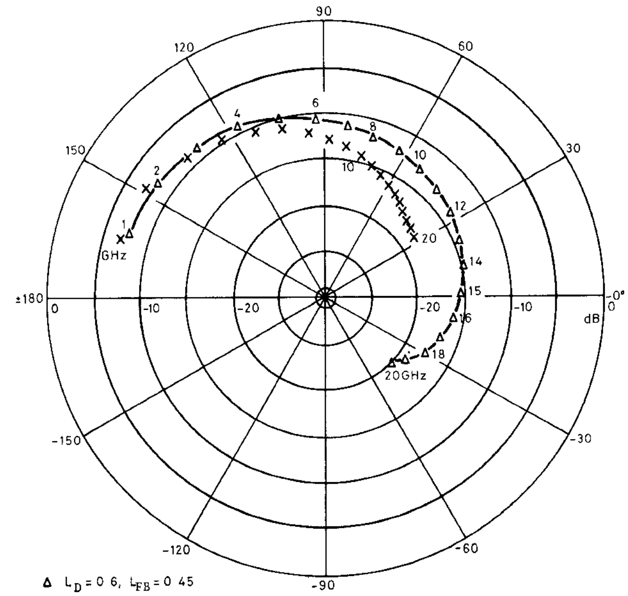

Fig. 2. Computed loop gains in beta path of circuit diagram (x case A, Δ case D, frequencies in gigahertz).

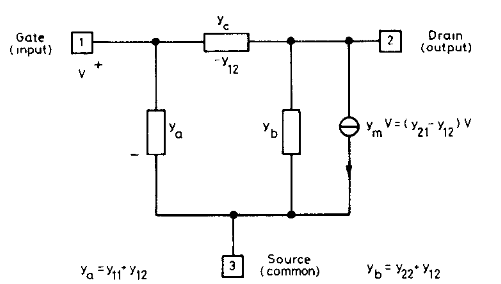

Fig. 3. Three-terminal admittance parameter representation of MESFET (values are computed from high-frequency model).

The most informative return ratio is that for the transadmittance $y_m = y_{21} - y_{12}$ of the very simple representation of Fig. 3 for the MESFET. This return ratio was computed for both case A and case D of [1], and the results are presented in Fig. 4. The

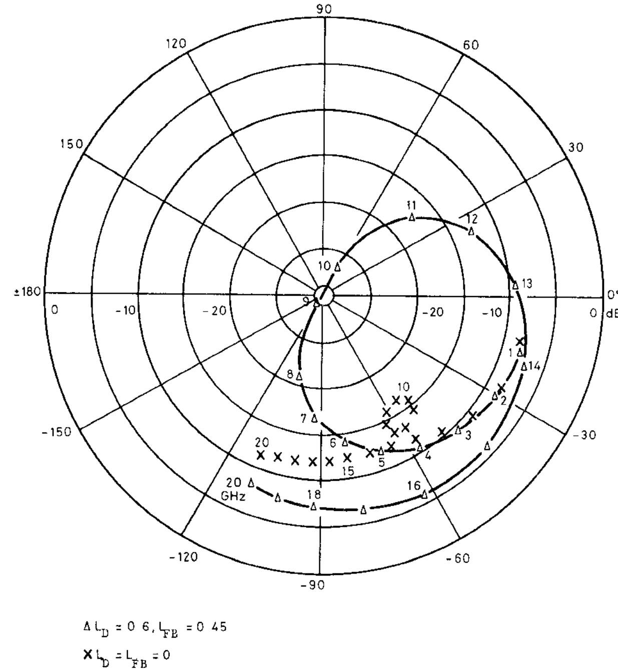

Fig. 4. Computed return ratios for y_m using circuit diagram for passive parts of Fig. 1(a) (frequencies in gigahertz).

difference between the two cases is much more marked than in Fig. 2. The magnitudes of these return ratios are, in general, larger than those of the corresponding loop gains. This occurs because the return ratio is a measure of total feedback, and not just that through the external path. Pure "positive feedback" corresponds to an angle of 180°, but the corresponding magnitude is around −27 dB. The gain margin of 27 dB is clearly much larger than the corresponding loop gain value (~15 dB). As before, a phase margin cannot be defined. The calculations showed that the largest amount of feedback (return difference) given by one plus the return ratio was only 2.86 dB at 1 GHz, an insignificant amount for most practical purposes. The amount of gain enhancement due to return differences less than 1 was also insignificant (0.18 and 0.15 dB at 8 and 9 GHz, respectively).

Although the present application of the embedding network method is to a rather artificial example, the method is applicable to amplifiers in the gigahertz range, and should yield much more realistic and reliable information for both the design and development of feedback amplifiers, even at these frequencies.

Reply[1] *by Karl B. Niclas, Walter T. Wilser, Richard B. Gold, and William R. Hitchens*[2]

There are a number of ways that can be used to understand the behavior of a feedback amplifier. The most common is probably that of loop gain plots as pointed out by Maclean in his comments on our paper [1].

The most common definition of gain that is used in the field of microwaves is that of insertion gain G, i.e., the gain that an amplifier provides when inserted between a 50-Ω source and a

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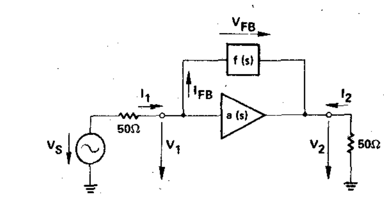

Fig. 5. Feedback amplifier in 50-Ω system.

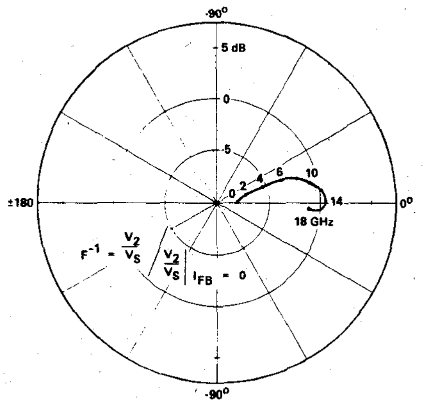

Fig. 6. Reciprocal of the return difference of the feedback amplifier described in fig. 1(a) of [2] as defined in (1c).

50-Ω load impedance (Fig. 5). In accordance with the definition of insertion gain, it seems to us that it is more practical to explain the behavior of a microwave feedback amplifier in terms of the source voltage V_s and the output voltage V_2 rather than input voltage V_1 and output voltage V_2 as is common in feedback theory [6].

On the basis of V_s and V_2, the open-loop transfer function is then defined as

$$a(s)=2\frac{V_2}{V_s}\bigg|_{I_{FB}=0} \tag{1a}$$

and the gain function of the closed-loop system becomes

$$A(s)=2\frac{V_2}{V_s} \tag{1b}$$

with the return difference changing to

$$F(s)=1-a(s)f(s)=\frac{\left.\frac{V_2}{V_s}\right|_{I_{FB}=0}}{\frac{V_2}{V_s}}. \tag{1c}$$

The magnitude and phase of $a(s)$ and $A(s)$ as defined in (1a) and (1b) are plotted in [2 fig. 5(b)] as curves A and C, respectively. The ratio of the magnitudes of curves A and C determines the return difference as defined by (1c). Its reciprocal, $F(s)^{-1}$, is shown in Fig. 6. At 2 GHz the open-loop gain is approximately 7.5 dB above that of the closed-loop system, which can hardly be called an insignificant amount. Due to the fact that $F(s)^{-1}$ is greater than one between 11 and 16.2 GHz, we have positive feedback, even though it is weak (Fig. 6). When operating in a 50 ohm system the return difference as commonly defined [6]

$$F(s)=\frac{\left.\frac{V_2}{V_1}\right|_{I_{FB}=0}}{\frac{V_2}{V_1}} \tag{2}$$

is less practical due to the frequency dependence of V_1. It is this dependence that leads us to the definitions (1a)–(1c) based on the source voltage V_s.

Finally, it should be pointed out that V_2 and V_1 are in phase at 13.725 GHz as we have determined by means of two different computing methods using COMPACT. This is in contrast to [1] in which the in-phase frequency was determined to be at "just above 15 GHz." However, the phase of V_2/V_s goes through 0° at 14.82 GHz, or just below 15 GHz.

REFERENCES

[1] K. B. Niclas, W. T. Wilser, R. B. Gold, W. R. Hitchins, "The matched feedback amplifier: Ultrawide-band microwave amplification with GaAs MESFET's," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 285–294, Apr. 1980.

[2] D. J. H. Maclean, "Practical analysis of wideband feedback amplifiers", *Proc. Inst. Elect. Eng.*, IEEE, Vol. 127, Pt. G, pp. 1–12, Feb. 1980.

[3] D. J. H. Maclean, "Improved assessment of stability margins in repeater amplifiers," *Frequenz*, vol. 34, no. 7, pp. 201–207 Jul. 1980.

[4] ——, "Improved methods of assessing feedback in wideband multiloop amplifiers," *IEEE Trans. Circuits Syst.*, vol. CAS-27, Sept. 1980.

[5] ——, "On the inadequacies of loop gain in broadband, multiloop feedback amplifiers," *IEEE Trans. Circuits Syst.*, vol. CAS-27, Sept. 1980.

[6] M. S. Ghausi, *Principles and Designs of Linear Active Circuits.* New York: McGraw-Hill, 1965, pp. 360–363.